# Tutorial: Introduction to Logical Geometry

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The central aim of Logical Geometry (LG) is to develop an interdisciplinary framework for the study of logical diagrams in the analysis of logical, linguistic and conceptual systems. Throughout history a variety of authors have constructed logical diagrams for analysing logical, linguistic and conceptual systems such as syllogistics, propositional logic, modal logic, generalised quantifiers, aspectual adverbs, colour concepts and metalogical concepts. Furthermore, on a more abstract level, several authors have studied logical and geometrical properties of various types of logical diagrams, such as the difference between Aristotelian and duality relations, the notion of Boolean closure and the relation between Aristotelian and Hasse diagrams. The first part of this tutorial will present an overview of the various ***applications in logic, philosophy, linguistics and artificial intelligence*** for which logical diagrams have been constructed in LG. In the second part of the tutorial, we discuss a number of ***abstract-logical topics*** related to logical diagrams, whereas in the third part, we focus on some more ***visual-geometric topics.*** In each of the three parts we will pay particular attention to the interdisciplinary variety of formal, empirical and historical perspectives adopted in LG.

As far as the ***logical*** applications of LG are concerned, we discuss the systems of modal logic (in particular, S5) and Public Announcement Logic. Among the ***linguistic*** applications we will briefly present the LG analysis of the subjective quantifiers *many* and *few*, gradable adjectives and definite descriptions. On a more ***conceptual*** level, LG has been applied to knowledge representation and AI, and to the perceptual field of colour theory. Underlying the analysis of this wide and interdisciplinary range of topics is the Boolean algebraic technique of bitstrings, which will be introduced at the end of the first part.

Concerning the ***abstract-logical*** properties of logical diagrams, LG first of all adopts an information-theoretic approach to corroborate the claim that the Aristotelian relations are hybrid between opposition relations and implication relations. A second crucial claim in LG concerns the logical independence of Aristotelian and duality relations, which relates to differences in logic-sensitivity between the two sets of relations. From the point of view of interdisciplinarity, the study of these abstract-logical topics integrates concepts from Boolean algebra, combinatorics, group theory and the philosophy of information.

As for the ***visual-geometric*** properties of logical diagrams, LG characterises the differences between Aristotelian and Hasse diagrams for Boolean algebras (in particular, B3 and B4) in terms of different vertex-first projections, both for 2D (hexagon) and for 3D (rhombic dodecahedron) visualisations. Furthermore, the difference between perspective and parallel projections leads to the 2D distinction between hexagons and nested triangles in the visualisation of B3, and to the 3D distinction between the rhombic dodecahedron and nested tetrahedra in the visualisation of B4. Other crucial geometric concepts studied in LG are those of distance, central symmetry and (the complementarities between) subdiagrams. With respect to interdisciplinarity, the LG analysis of these visual-geometric topics not only employs concepts from Boolean algebra and combinatorics, but also from diagram design. More in particular, logical diagrams are argued to differ in terms of informational versus computational equivalence, and in terms of their adherence to – or violation of – such design principles as congruence and apprehension.

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