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Logical and Geometrical Complementarities between Aristotelian Diagrams

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central idea: **subdiagrams** / diagram embedding / diagram nesting

- smaller diagrams occur inside bigger diagrams
- Euler diagrams (Fish & Flower 2008), Venn diagrams (Flower, Stapelton & Rogers 2013), spider diagrams (Urbas, Jamnik, Stapelton & Flower 2012) or algebra diagrams (Cheng 2012)
- visual nesting and recursion (Engelhardt 2006): "*any object may contain a set of (sub-)objects within the space that it occupies. When this principle is repeated recursively, the spatial arrangement of (sub-)objects is, at each level, determined by the specific nature of the containing space at that level.*"
- set of transitively nested substates of a composite state (Jin, Esser & Janneck 2002): "*in every UML statechart there is an inherent composite state called the **top state** which covers all the (pseudo) states and is the container of the states.*"

central diagram: **rhombic dodecahedron (RDH)**

- Smessaert (2009/2012), Demey (2012)
- 3D representation for the visualisation of the Aristotelian relations between 14 contingent formulas.

central aims of talk

- to develop strategies for systematically charting the internal structure of the RDH
- to study various complementarities between Aristotelian diagrams inside the RDH
- to provide a more unified account of a whole range of diagrams which have so far mostly been treated independently of one another in the literature

Overview

- Informally, two formulas are:

contradictory	iff they cannot be true together and cannot be false together
contrary	iff they cannot be true together, but can be false together
subcontrary	iff they can be true together, but cannot be false together
in ***subalternation***	iff the first logically entails the second, but not vice versa

- Formally (relative to a logical system S), two formulas φ, ψ are

contradictory	iff	$\mathsf{S} \models \neg(\varphi \wedge \psi)$	and	$\mathsf{S} \models \neg(\neg\varphi \wedge \neg\psi)$
contrary	iff	$\mathsf{S} \models \neg(\varphi \wedge \psi)$	and	$\mathsf{S} \not\models \neg(\neg\varphi \wedge \neg\psi)$
subcontrary	iff	$\mathsf{S} \not\models \neg(\varphi \wedge \psi)$	and	$\mathsf{S} \models \neg(\neg\varphi \wedge \neg\psi)$
in ***subalternation***	iff	$\mathsf{S} \models \varphi \rightarrow \psi$	and	$\mathsf{S} \not\models \psi \rightarrow \varphi$

- Formulas which do **not** stand in any Aristotelian relation are said to be ***unconnected*** or ***logically independent***.

Partitioning the formulas from Classical Propositional Logic with two propositional variables p and q into **Pairs of Contradictories (PCDs)**

- 4 PCDs of **type C** ('cube')

a.	$(p \wedge q)$	b.	$\neg(p \rightarrow q)$	c.	$\neg(p \leftarrow q)$	d.	$\neg(p \vee q)$
a'.	$\neg(p \wedge q)$	b'.	$(p \rightarrow q)$	c'.	$(p \leftarrow q)$	d'.	$(p \vee q)$

- 3 PCDs of **type O** ('octahedron') + 1 non-contingent PCD

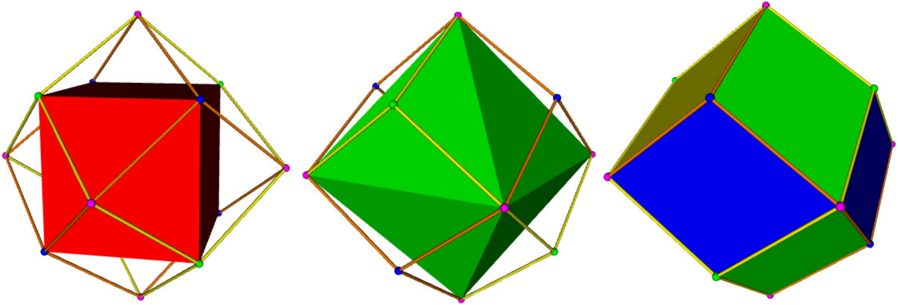
e.	p	f.	q	g.	$(p \leftrightarrow q)$	h.	$p \wedge \neg p$
e'.	$\neg p$	f'.	$\neg q$	g'.	$\neg(p \leftrightarrow q)$	h'.	$p \vee \neg p$

- Distinction between **levels**:

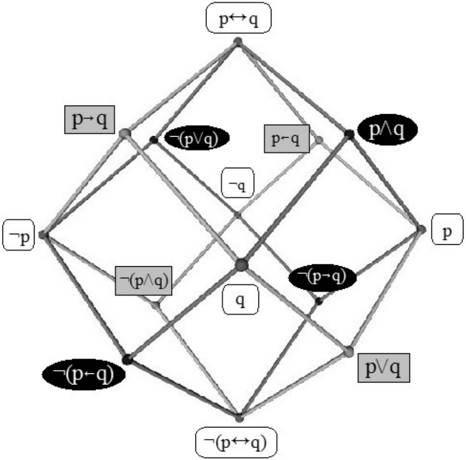
(a-d)	=	Level 1	$\Rightarrow$	pairwise ***contraries***
(a'-d')	=	Level 3	$\Rightarrow$	pairwise ***subcontraries***
(e-g) + (e'-g')	=	Level 2	$\Rightarrow$	pairwise ***unconnected***
(h-h')	=	Level 0/4	$\Rightarrow$	disregarded!

- 14 formulas/7 PCDs $\Rightarrow$ 3D visualisation

cube	+	octahedron	=	cuboctahedron	$\stackrel{dual}{\Longrightarrow}$	**rhombic dodecahedron**
Platonic		Platonic		Archimedean		Catalan
6 faces		8 faces		14 faces		**12 faces**
8 vertices		6 vertices		12 vertices		**14 vertices**



central symmetry	cube	=	4 PCDs of type C (L1-L3)
	octahedron	=	3 PCDs of type O (L2-L2)



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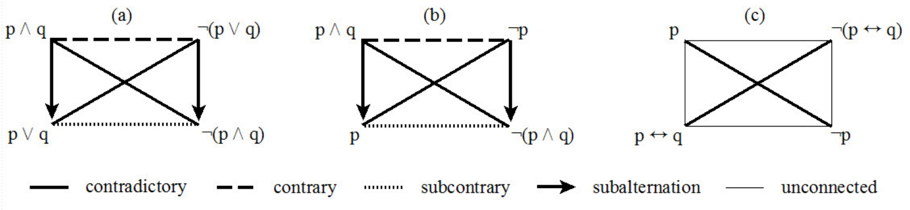
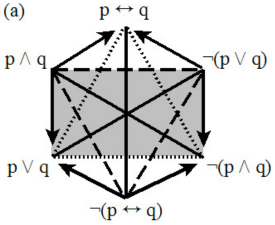
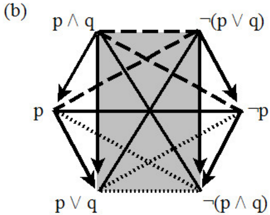
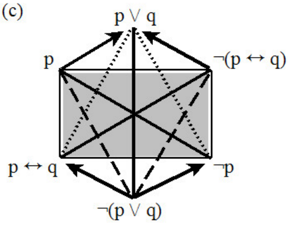


classical	classical	degenerate
balanced	unbalanced	balanced
C_2O_0	C_1O_1	C_0O_2
L1-L3 + L1-L3	L1-L3 + L2-L2	L2-L2 + L2-L2





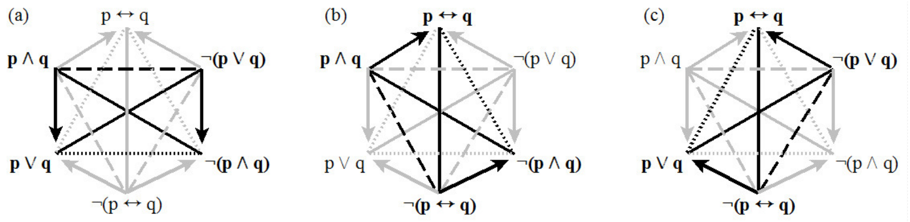


Jacoby-Sesmat-Blanché (JSB)
C_2O_1a
L1-L3/L1-L3+**L2-L2**
triangles of (sub)contrariety

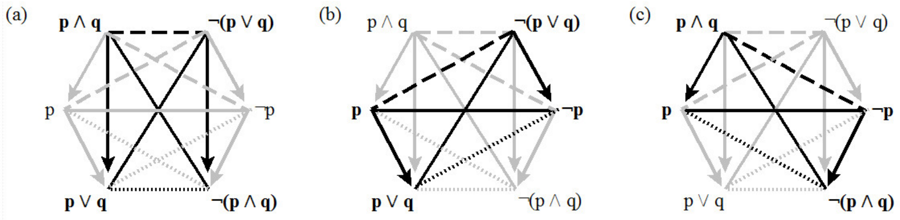
Sherwood-Czezowski (SC)
C_2O_1b
L1-L3/L1-L3+**L2-L2**
triangles of subalternation

Unconnected-4 (U4)
C_1O_2
L2-L2/L2-L2+**L1-L3**
no triangles
$\rightarrow$ V-shapes

(cfr. Smessaert, *Diagrams 2012*)



squares in Jacoby-Sesmat-Blanché hexagon (JSB)



squares in Sherwood-Czezowski hexagon (SC)

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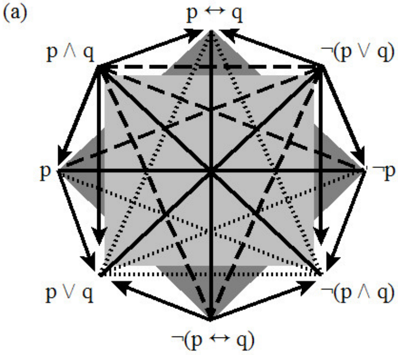
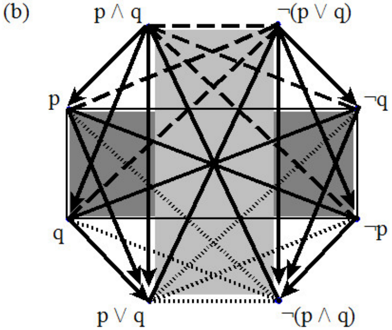


Béziau octagon

C_2O_2b

2 triangles of (sub)contrariety

2 triangles of subalternation

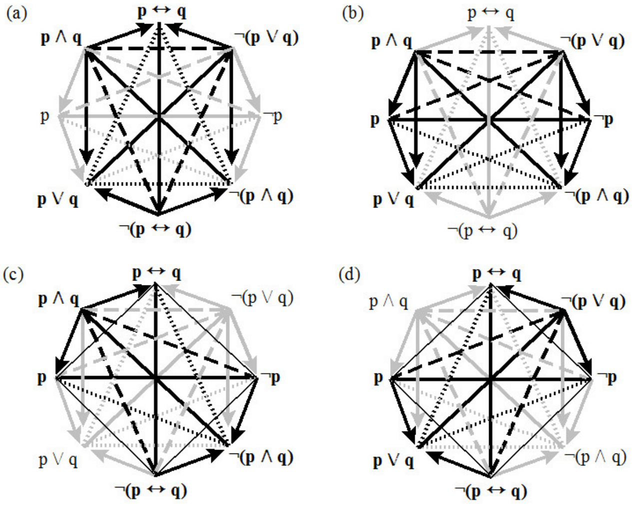


Buridan octagon

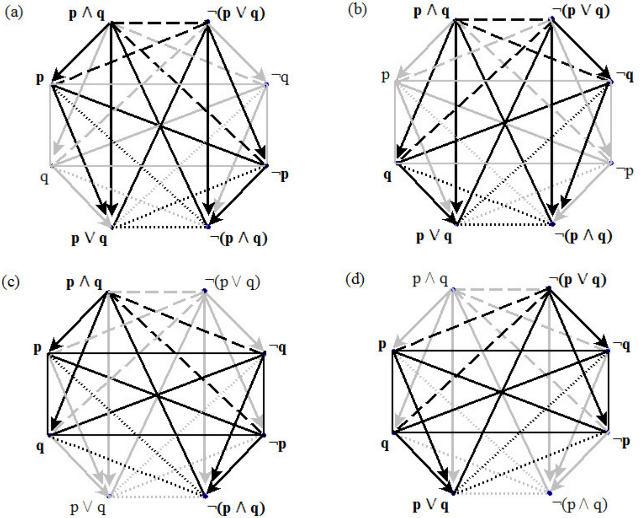
C_2O_2a

0 triangles of (sub)contrariety

4 triangles of subalternation



(a) JSB (b) SC (c) U4 (d) U4

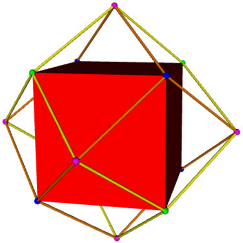


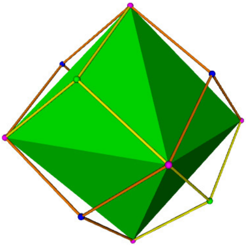
(a) SC (b) SC (c) U4 (d) U4

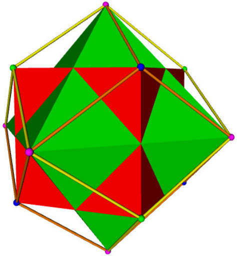
Complementarities in the Rhombic Dodecahedron

Fundamental complementarity: $C_k O_\ell + C_{4-k} O_{3-\ell} = C_4 O_3 = RDH$

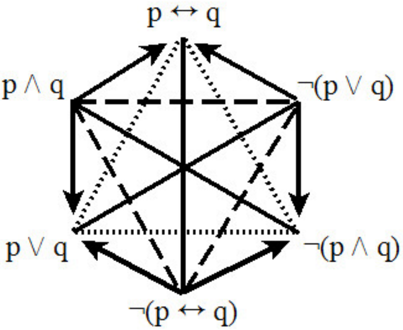
C_4O_0	$+$	C_0O_3	$=$	C_4O_3
cube	+	octahedron	=	RDH

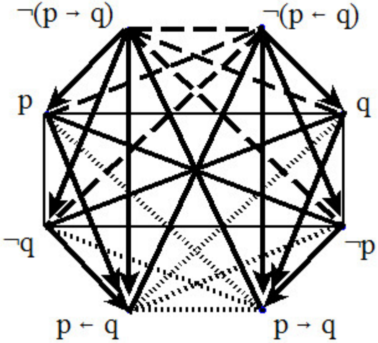


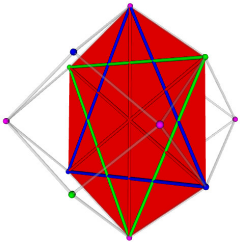
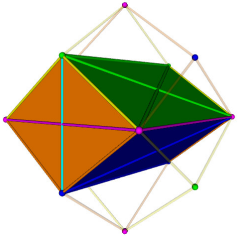
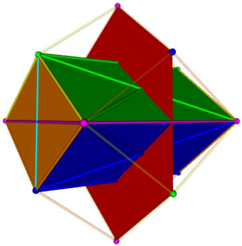




C_2O_1a	+	C_2O_2a
Jacoby-Sesmat-Blanché hexagon	+	Buridan octagon (2 SC hexagons)





C_2O_1a	+	C_2O_2a	=	C_4O_3
Jacoby-Sesmat-Blanché hexagon	+	Buridan octagon	=	rhombic dodecahedron
				
hexagonal plane	+	**rhombicube**	=	RDH

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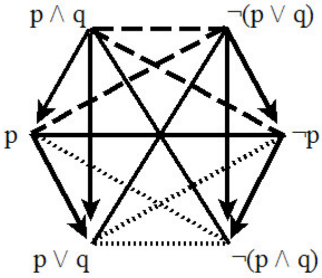
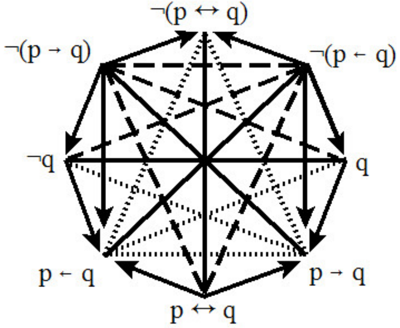
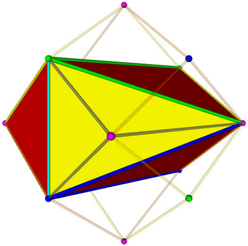
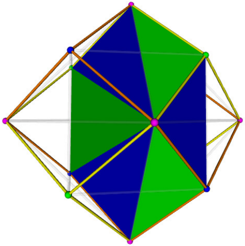
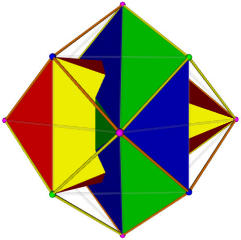
C_2O_1b	+	C_2O_2b
Sherwood-Czezowski hexagon	+	Béziau octagon
		(1 JSB + 1 SC hexagon)





C_2O_1b	+	C_2O_2b	=	C_4O_3
Sherwood-Czezowski hexagon	+	Béziau octagon	=	rhombic dodecahedron
				
squeezed octahedron	+	**squeezed hexa-gonal bipyramid**	=	RDH

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develop strategies for systematically charting the internal structure of RDH

- distinguish families of *squares* in terms of 2 embedded *PCDs*
- distinguish families of *hexagons* in terms of 3 embedded *squares*
- distinguish families of *octagons* in terms of 4 embedded *hexagons*

study various complementarities between Aristotelian diagrams inside RDH

- difference in visual appeal $\approx$ geometric difference (reflection symmetries)
- SC hexagons naturally come in pairs (Buridan octagon/rhombicube)
- JSB-Buridan complementarity respects this pairing
- SC-Béziau complementarity cuts across this pairing

provide a more unified account of a whole range of diagrams which have so far mostly been treated independently of one another in the literature

$\Rightarrow$ exhaustive typology of RDH subdiagrams (combinatorial analysis)

Thank you!

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