Category Theory and Logical Geometry – Is a commutative diagram an Aristotelian diagram?

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Category theory is concerned with structural equivalence between different objects in the same and between different formal frameworks (categories). Its most important tool are commutative diagrams (structure-preserving arrow-diagrams), which serve (as the drawings in classical geometry) as the foundation for a special kind of diagraphical reasoning. In its form of topos theory it is powerful enough to provide an analysis and a reconstruction of classical-mathematical and intuitionist logic. My presentation revolves around the question "Are the diagrams of category theory a kind of Aristotelian diagram?"

One of the main selling points of category theory is that it provides a macro view on formal structures and their inter-relations that would not be possible by any "direct" comparison of these structures. The same is true of Aristotelian diagrams. Both provide an overview by representing structures and their relations by a picture. In both cases, this picture can, according to a special kind of grammar, get a step-by-step interpretation. But this pictorial mode of representing is itself not sequential and the information it contains cannot be reduced to one sequential reading. Furthermore, the geometrical features of the diagrams can "show" new ways how to read (or sometimes to rearrange) it.

In my presentation, I will give a short introduction into the way diagrams are utilized in category theory. I will — by using a simple example — show how the contemplation of such a diagram can trigger the discovery of new inter-structural features and point in a way how to prove it. I will compare this to simple examples of classical Aristotelian diagrams and the way they are utilized. In this comparison the focus will be on the function of the geometrical property of symmetry.